Slutsky and perfect substitutes

Given the following utility function $u = x_1 + x_2$

1. Assume that initially $m = 10$, $p_1 = 1$, $p_2 = 5$. Later, the price of good 1 becomes $p_1' = 2$. Find the total effect, income effect, and substitution effect for both goods.
2. Now assume that the price of good 1 is $p_1' = 10$. Find the total effect, income effect, and substitution effect for both goods.

Solutions

1. With these prices, the individual demands only x_1, the optimal basket is $(10/1, 0)$. If the price increases to 2, then the individual continues to demand only x_1 since $p_1 < p_2$ still holds. But now the demand is lower: $(10/2, 0) = (5, 0)$. This happens due to a decrease in the individual's purchasing power; the total effect is equal to the income effect. For good 2, there is no effect as the demand did not change.

2. If the price changes to $p_1' = 10$, then the individual will demand only x_2, the optimal basket will be $(0, 10/5) = (0, 2)$. Therefore, now the individual demands 10 less of x_1, and this is due to substitution; there is no income effect. On the other hand, for good x_2, 2 more are demanded than before. The total effect is 2. To find the income and substitution effects, we first find the Hicksian demands; for that, we first invert the indirect utility function to get the expenditure function:

$$V = M/p_1$$

$$E = \bar{u}p_1$$

$$E'p_1 = x_1^h = \bar{u}$$

$$E'p_2 = x_2^h = 0$$

While when demanding only x_2,

$$V = M/p_2$$

$$E = \bar{u}p_2$$

$$E'p_1 = x_1^h = 0$$

$$E'p_2 = x_2^h = \bar{u}$$

Therefore, the increase in demand for x_2, given by the substitution effect, is:

$$\bar{u} - 0 = 10 - 0 = 10$$

And the income effect is the difference between the total effect and the substitution effect:

$$TE - SE = IE = 2 - 10 = -8$$